

Supplementary Material for *A Wide-Ranging  
Computational Comparison of High-Performance  
Graph Colouring Algorithms*

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## Abstract

This document contains a more comprehensive set of graphs describing the behaviour of the various algorithms in experiments described in *A Wide-Ranging Computational Comparison of High-Performance Graph Colouring Algorithms*. Details are also provided on how to reproduce the problems considered in the study. All of the algorithm implementations used in the study (in c++) can be downloaded, together with instructions, from <http://www.rhydlewislewis.eu/resources/gCol.zip>.

# Chapter 1

## Random Graphs

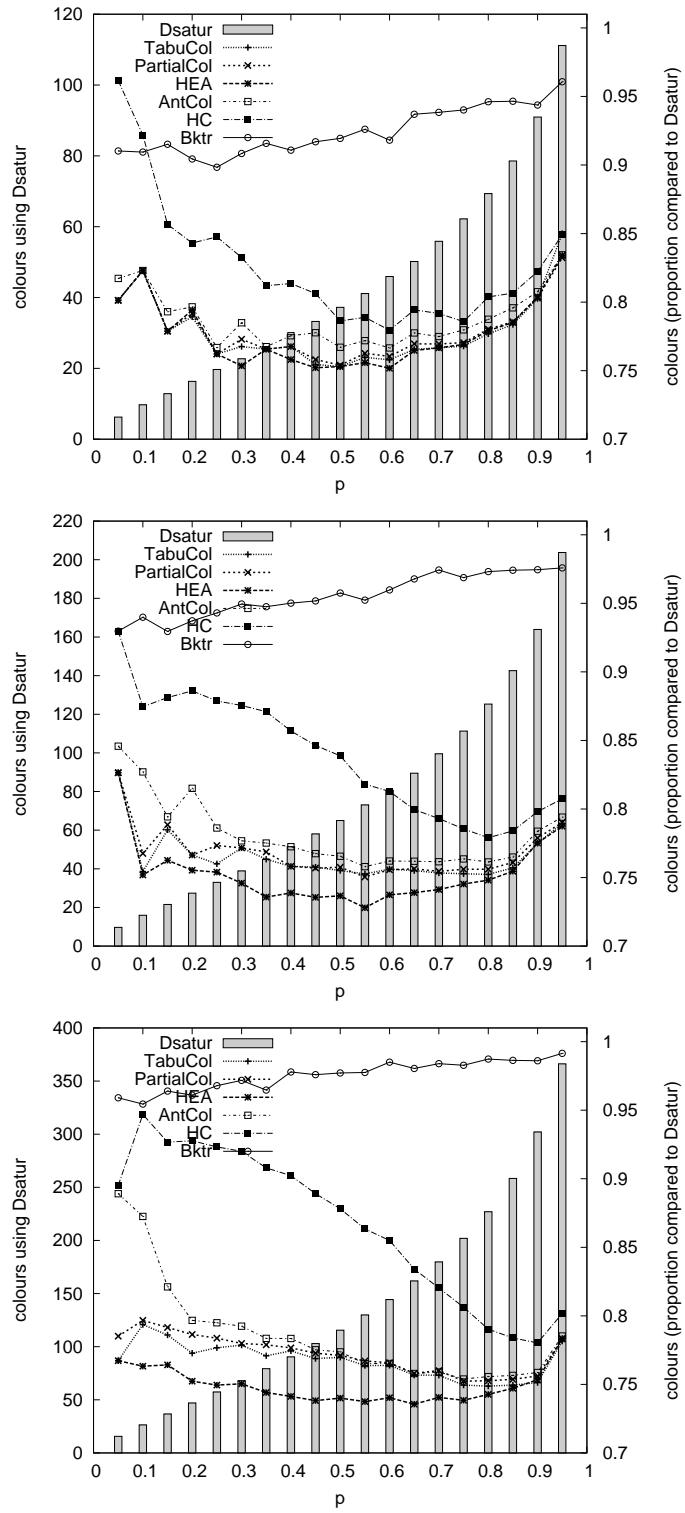


Figure 1.1: Summary of mean algorithm performance for random graphs of sizes  $|V| = 250$ , 500, and 1000 respectively.

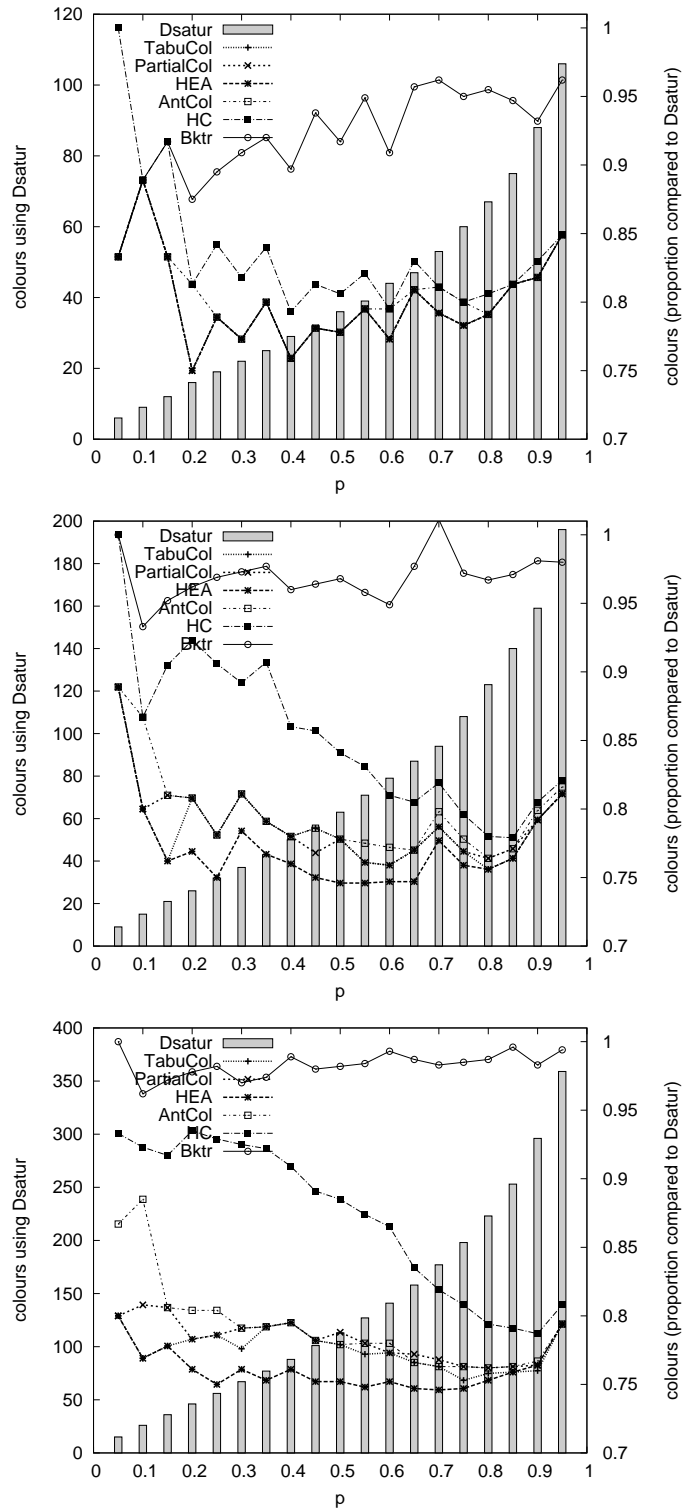


Figure 1.2: Summary of best results from each algorithm on random graphs of sizes  $|V| = 250, 500, \text{ and } 1000$  respectively.

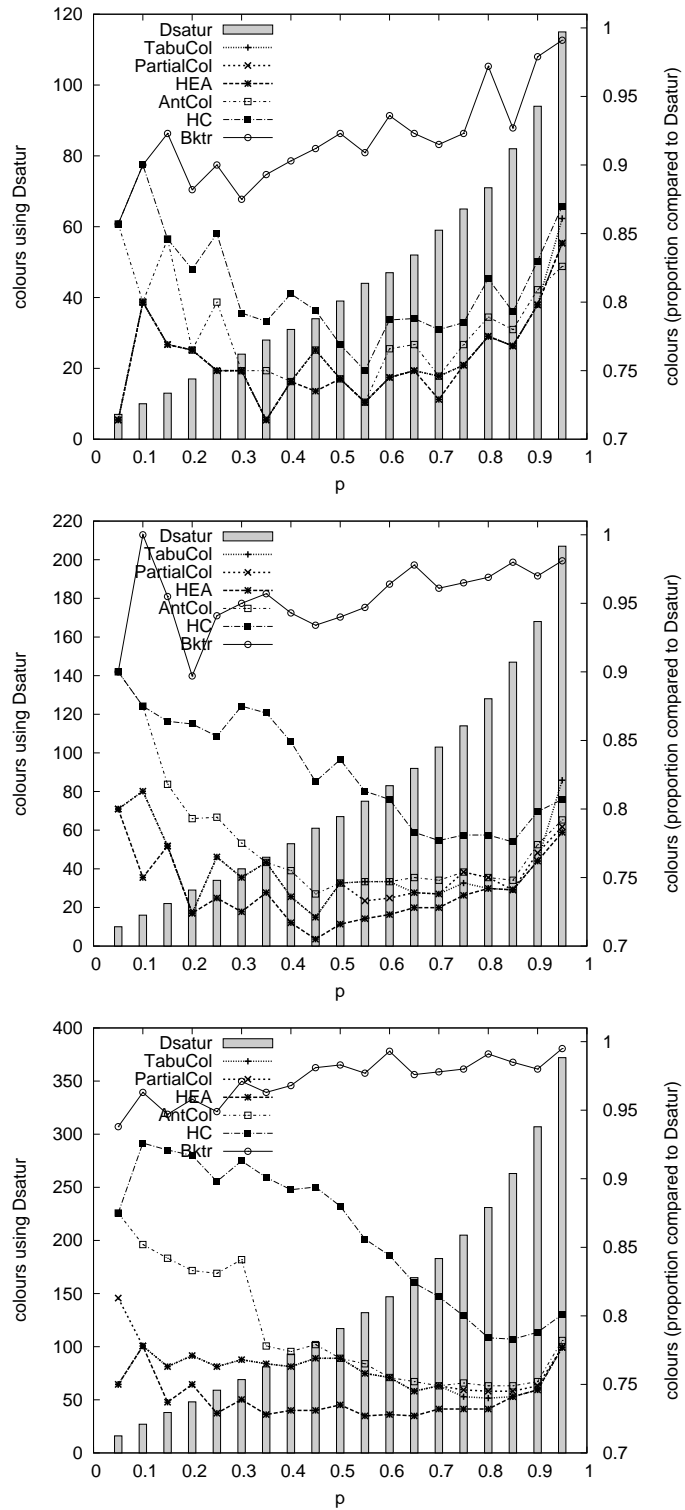


Figure 1.3: Summary of worst results from each algorithm on random graphs of sizes  $|V| = 250, 500, \text{ and } 1000$  respectively.

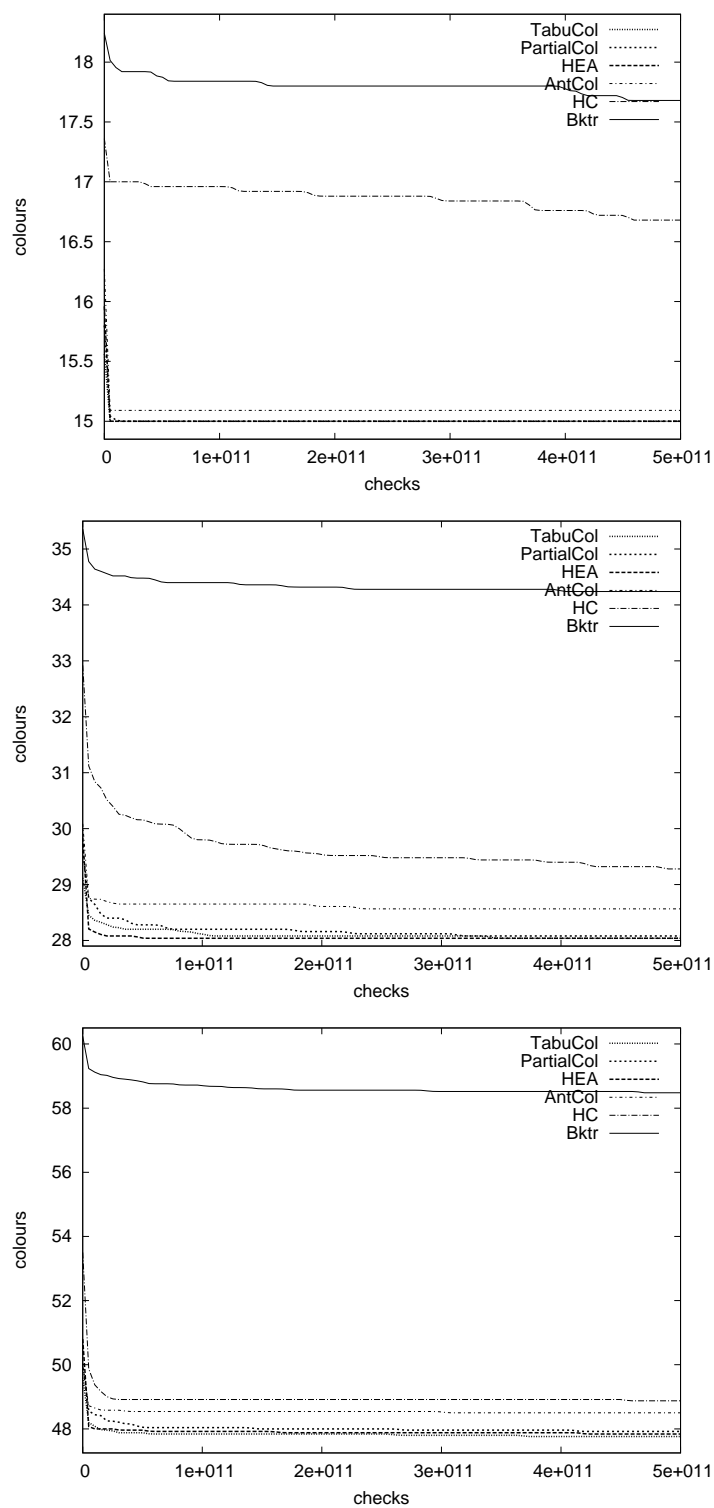


Figure 1.4: Run profiles for random graphs of sizes  $|V| = 250$  with  $p = 0.25, 0.5,$  and  $0.75$  respectively.

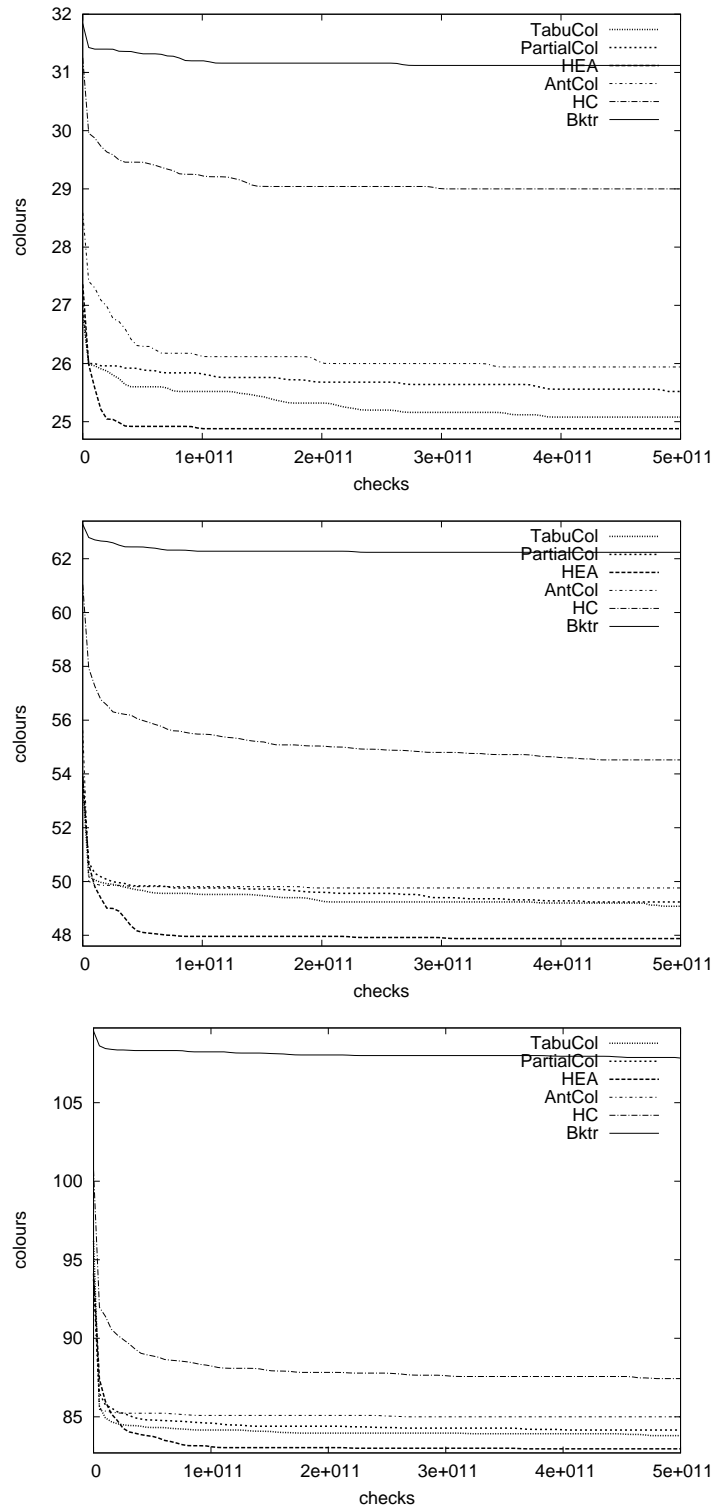


Figure 1.5: Run profiles for random graphs of sizes  $|V| = 500$  with  $p = 0.25, 0.5,$  and  $0.75$  respectively.



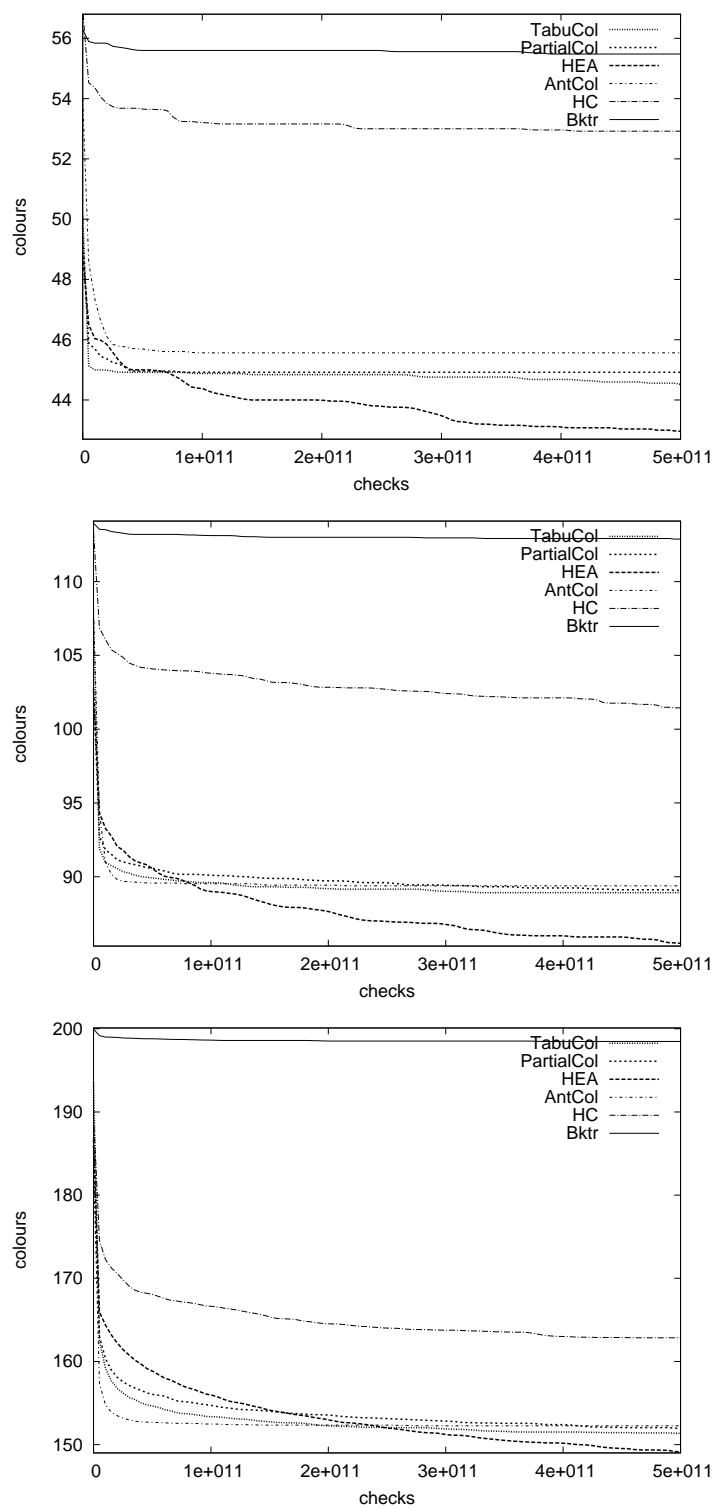


Figure 1.6: Run profiles for random graphs of sizes  $|V| = 1000$  with  $p = 0.25, 0.5,$  and  $0.75$  respectively.

## Chapter 2

# Flat Graphs

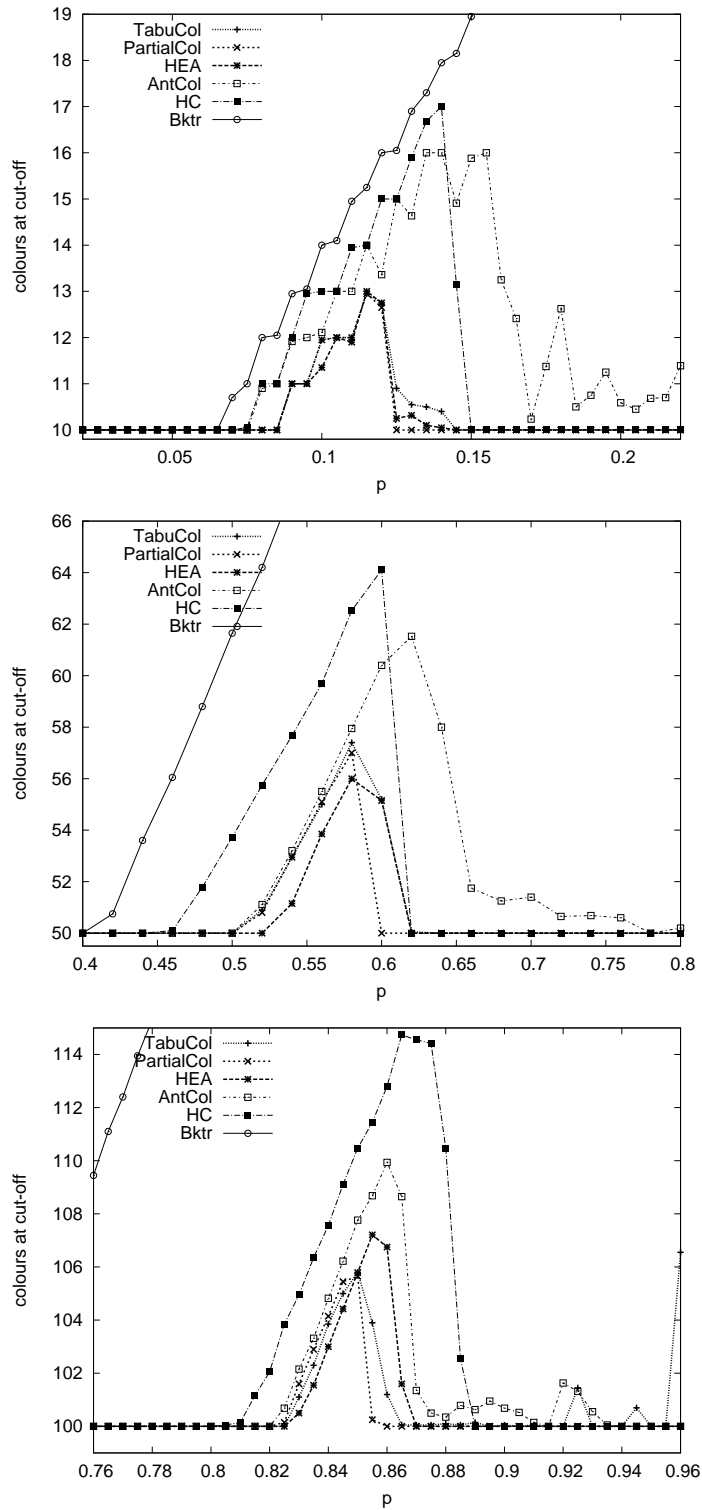


Figure 2.1: Summary of mean algorithm performance with flat graphs of size  $|V| = 500$ , with  $k = 10, 50$  and  $100$  respectively

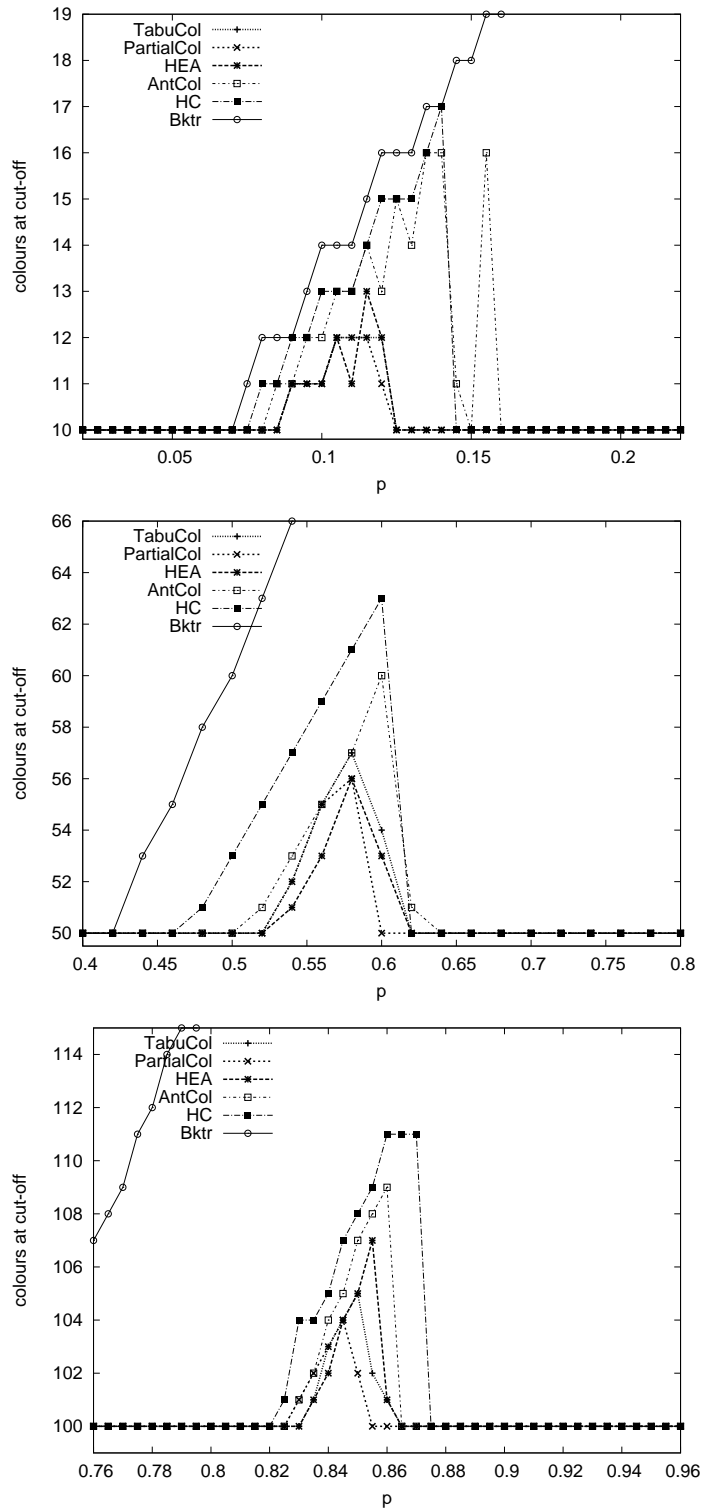


Figure 2.2: Summary of best results of each algorithm with flat graphs of size  $|V| = 500$ , with  $k = 10, 50$  and  $100$  respectively

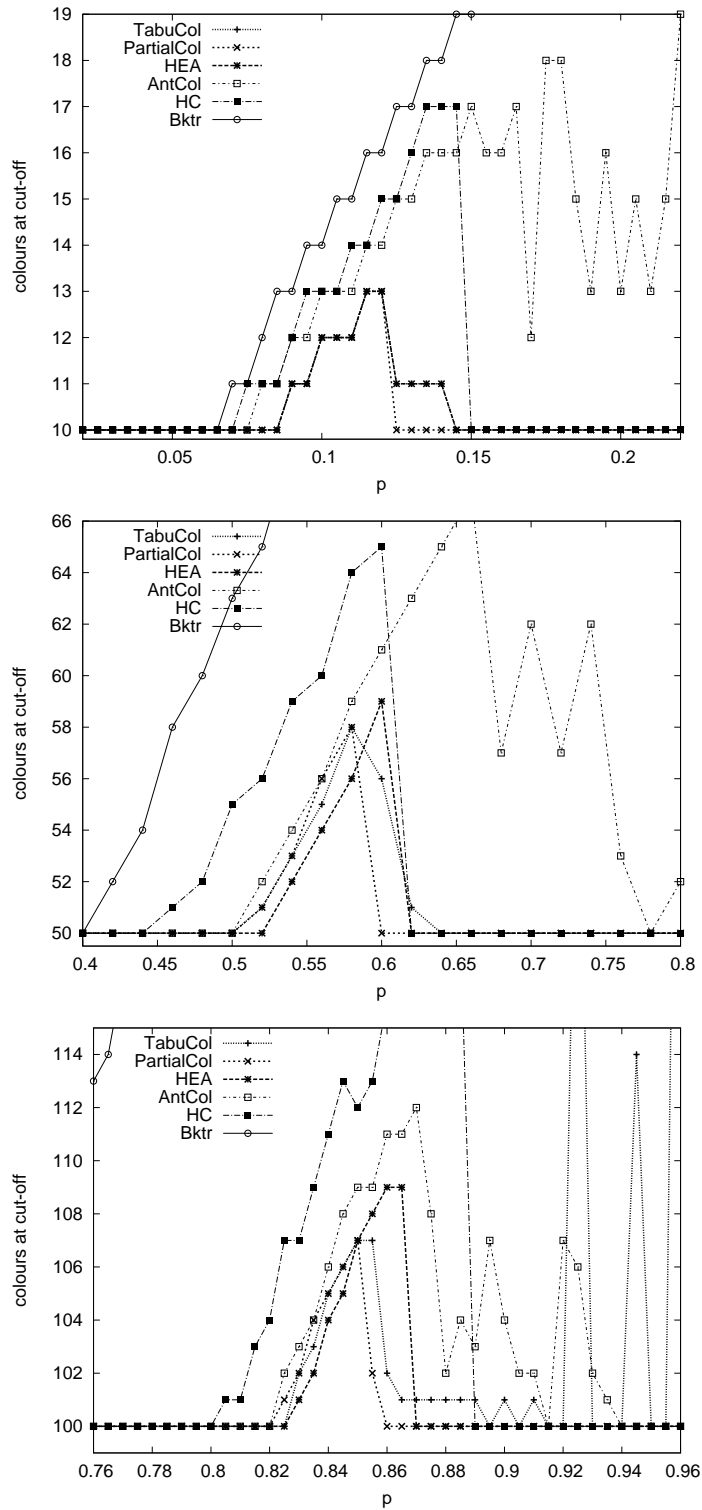


Figure 2.3: Summary of best results of each algorithm with flat graphs of size  $|V| = 500$ , with  $k = 10, 50$  and  $100$  respectively

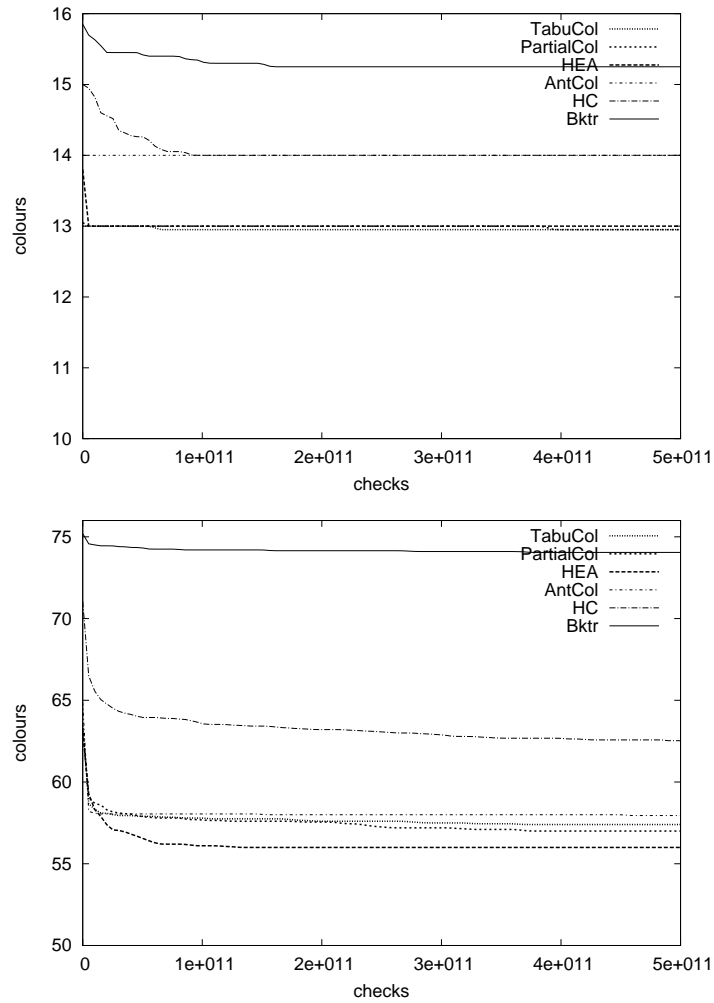


Figure 2.4: Run profiles for flat graphs with  $k = 10$ ,  $p = 0.115$  and  $k = 50$ ,  $p = 0.58$  respectively.

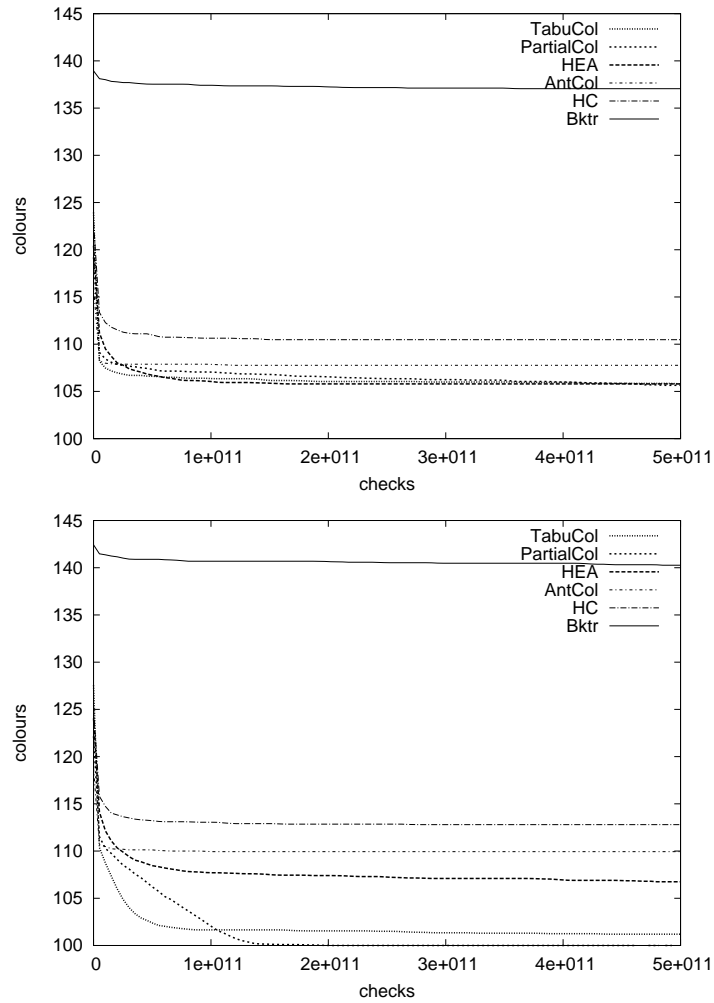


Figure 2.5: Run profiles for flat graphs with  $k = 100, p = 0.85$  and  $k = 100, p = 0.86$  respectively.

## Chapter 3

# Timetabling Graphs



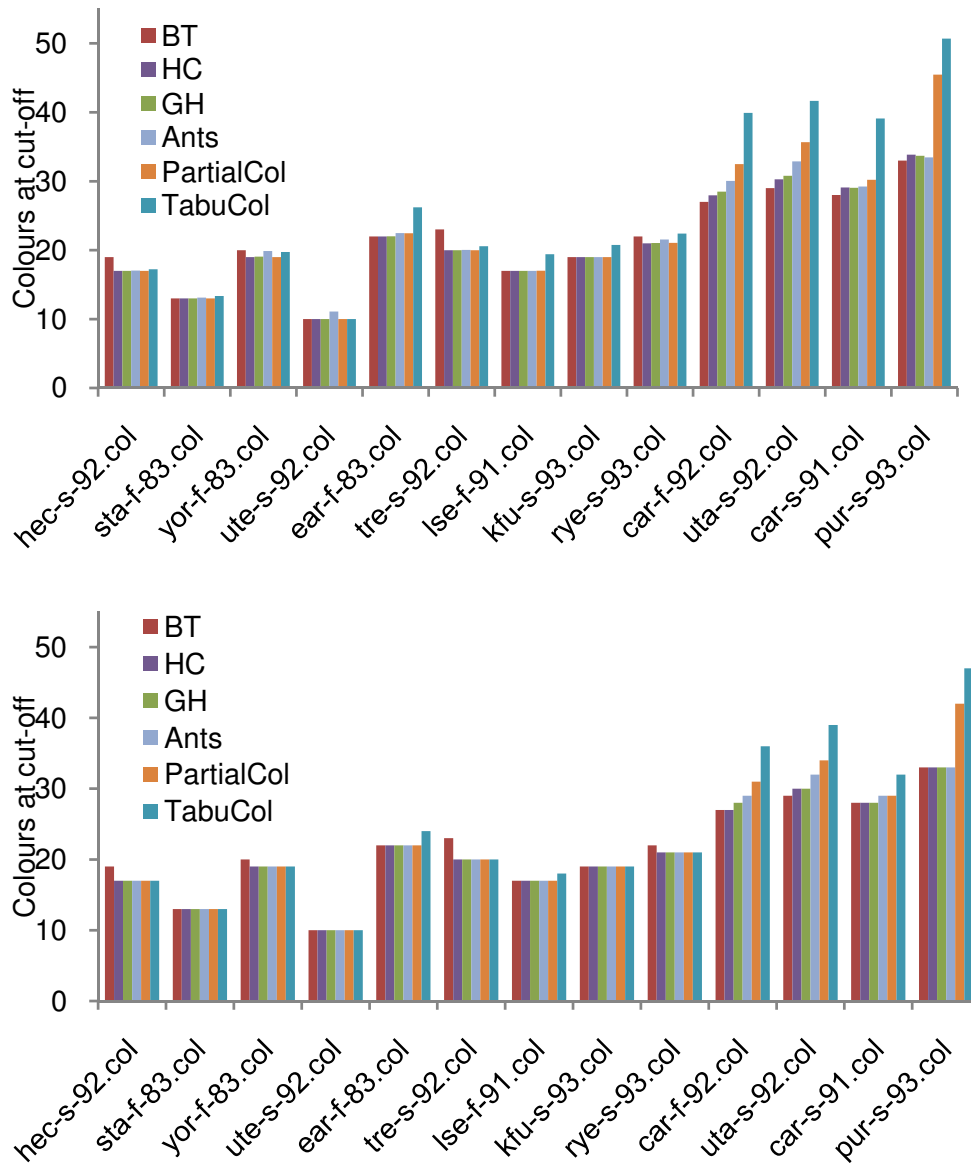


Figure 3.1: Mean (top) and best (bottom) results with the 13 Carter Instances.

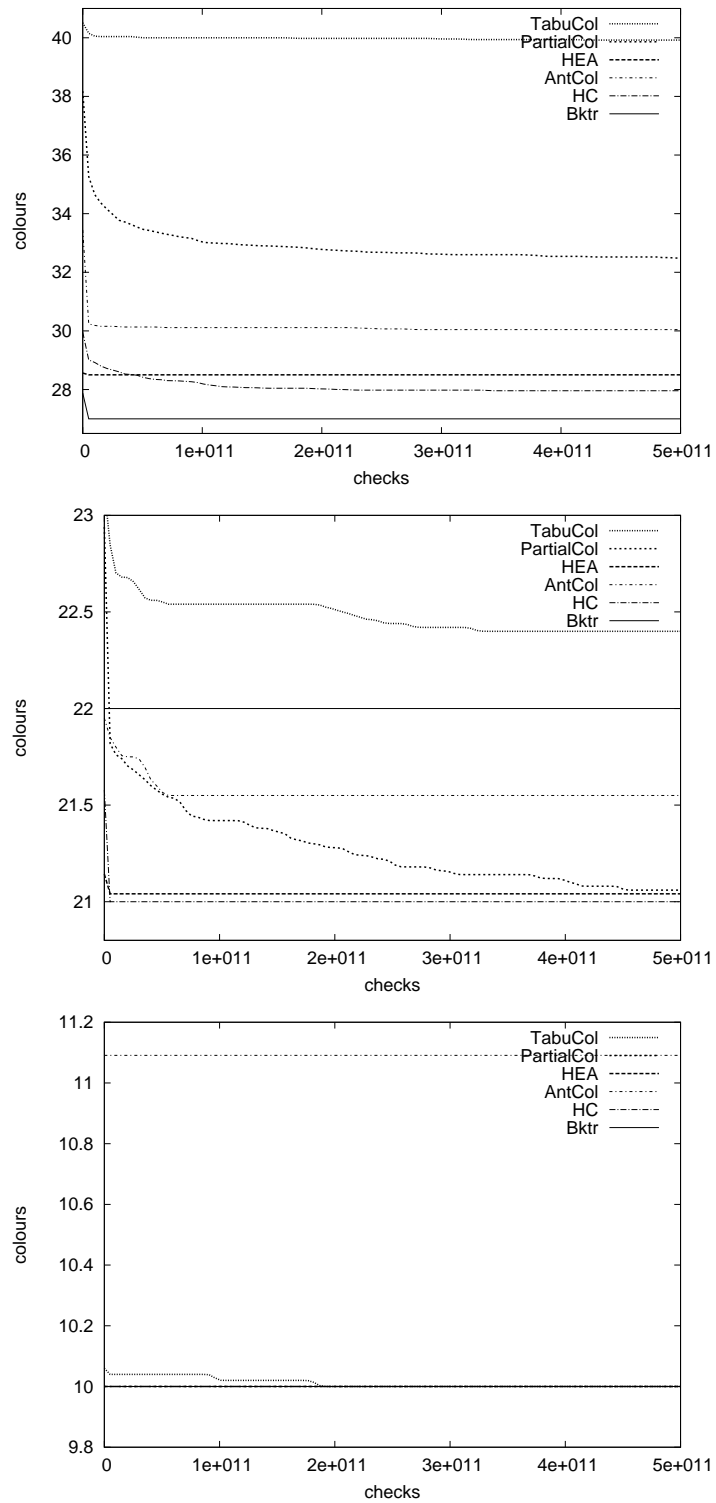


Figure 3.2: Run profiles for timetabling instances car-f-92, rye-s-93 and ute-s-92 respectively.

# Chapter 4

## Social Networks

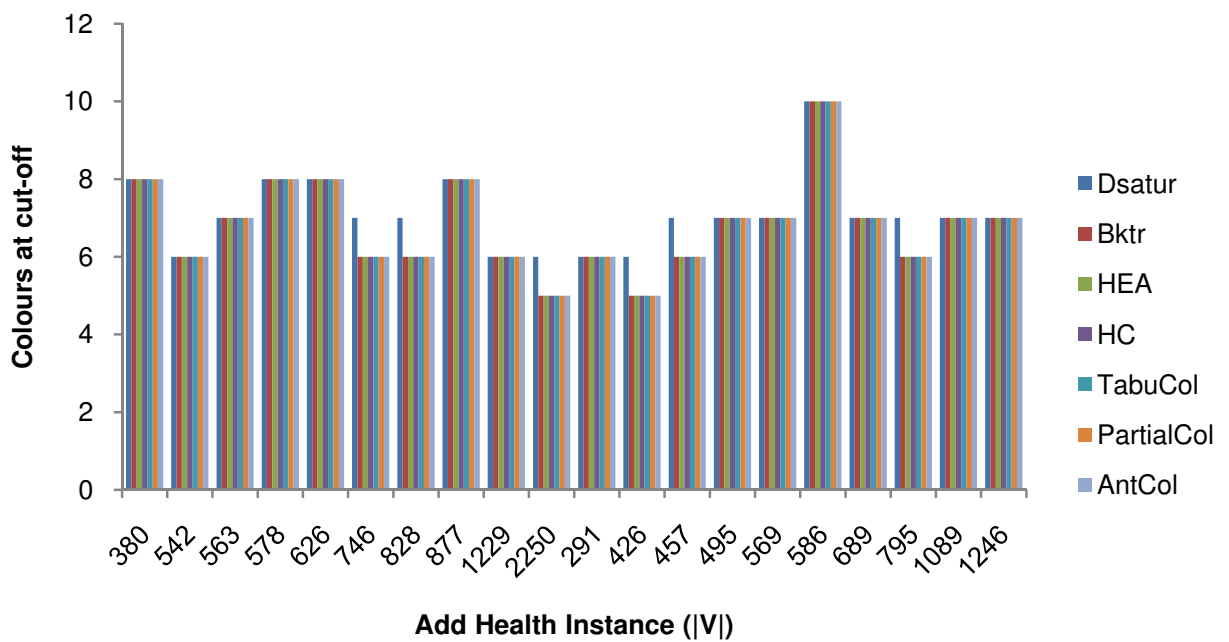
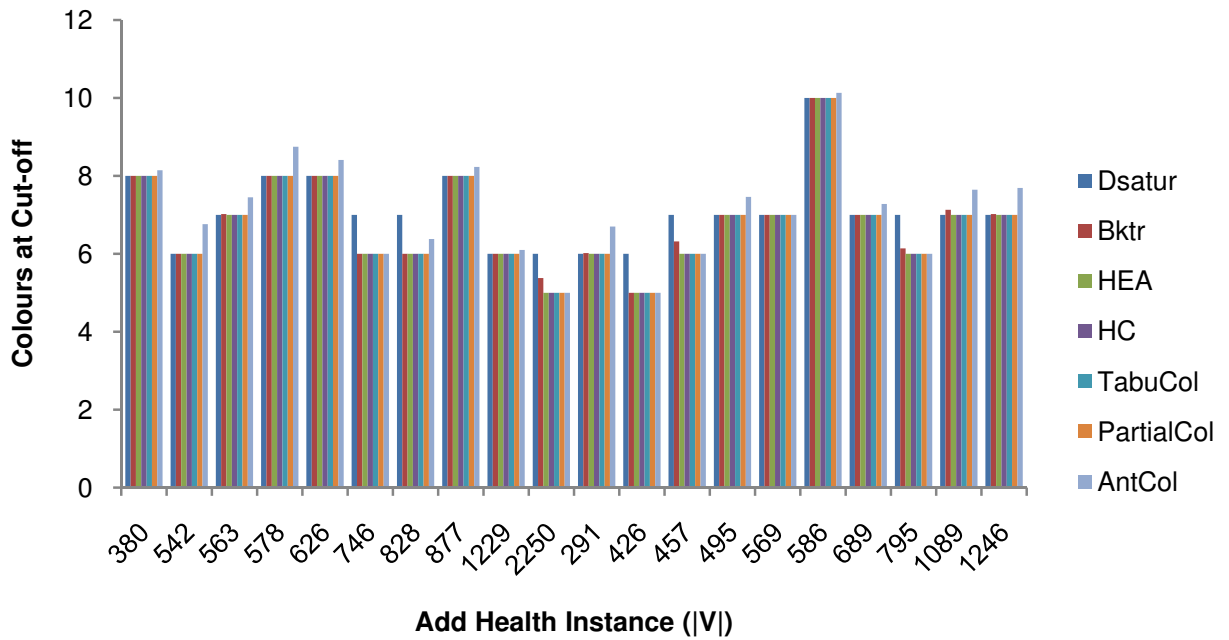


Figure 4.1: Mean (top) and best (bottom) results with the sample of 20 Social Networks. (The first 10 are single cluster instances (ordered by size); the second 10 are double-cluster).

# Chapter 5

## Round-Robin Graphs

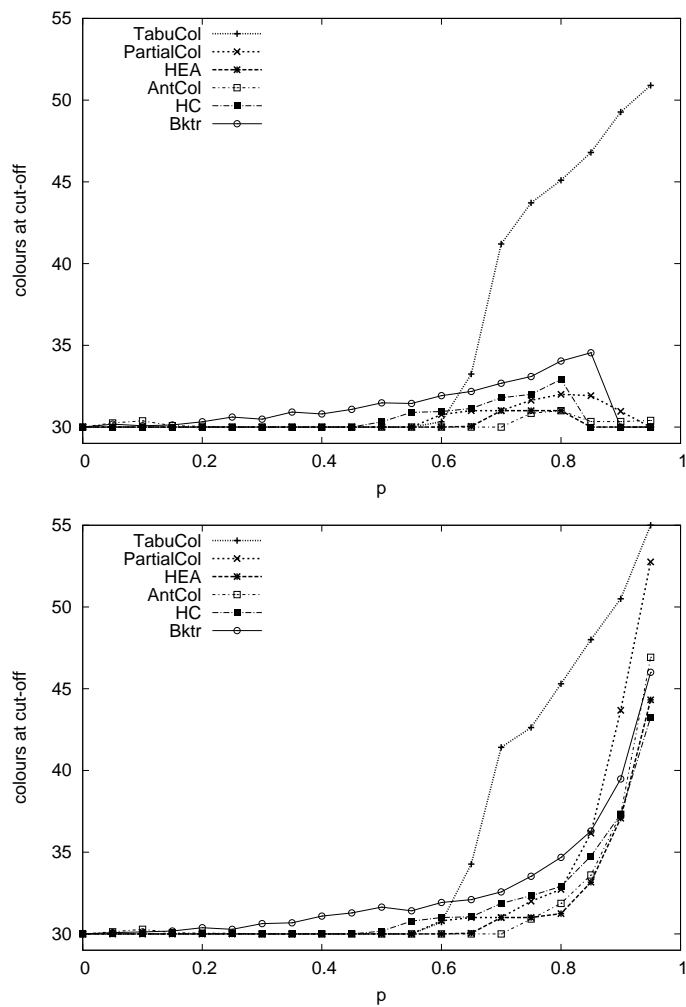


Figure 5.1: Summary of mean algorithm performance with (top) “solvable” round-robin graphs of size  $|V| = 270$ , with  $\chi = 30$ ; and (bottom) round-robin graphs of size  $|V| = 270$ , with  $\chi \geq 30$ .

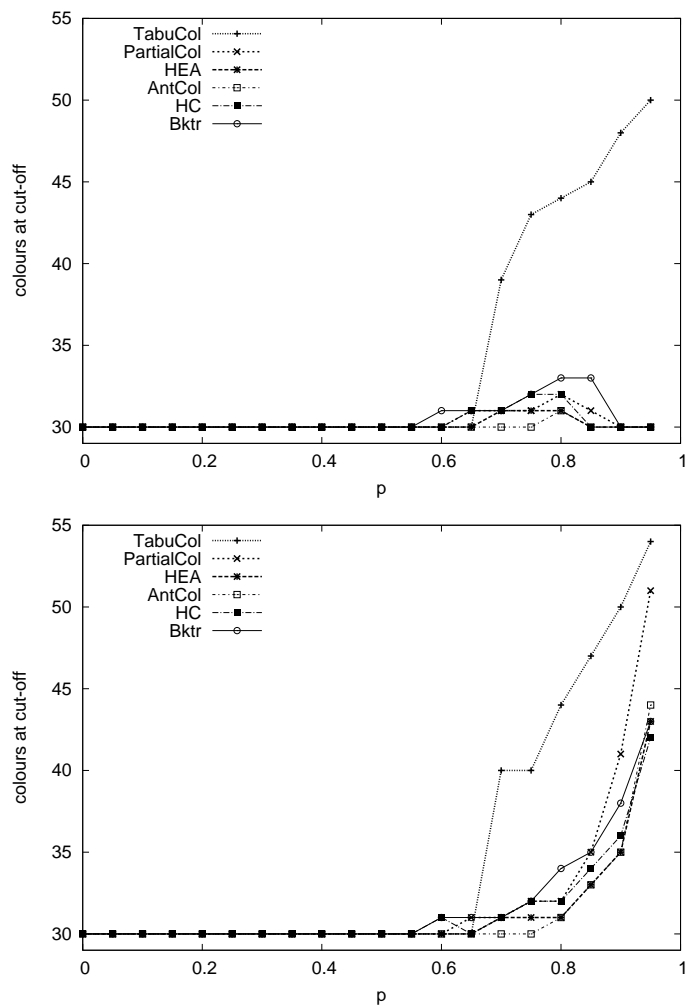


Figure 5.2: Summary of best results from each algorithm with (top) “solvable” round-robin graphs of size  $|V| = 270$ , with  $\chi = 30$ ; and (bottom) round-robin graphs of size  $|V| = 270$ , with  $\chi \geq 30$ .

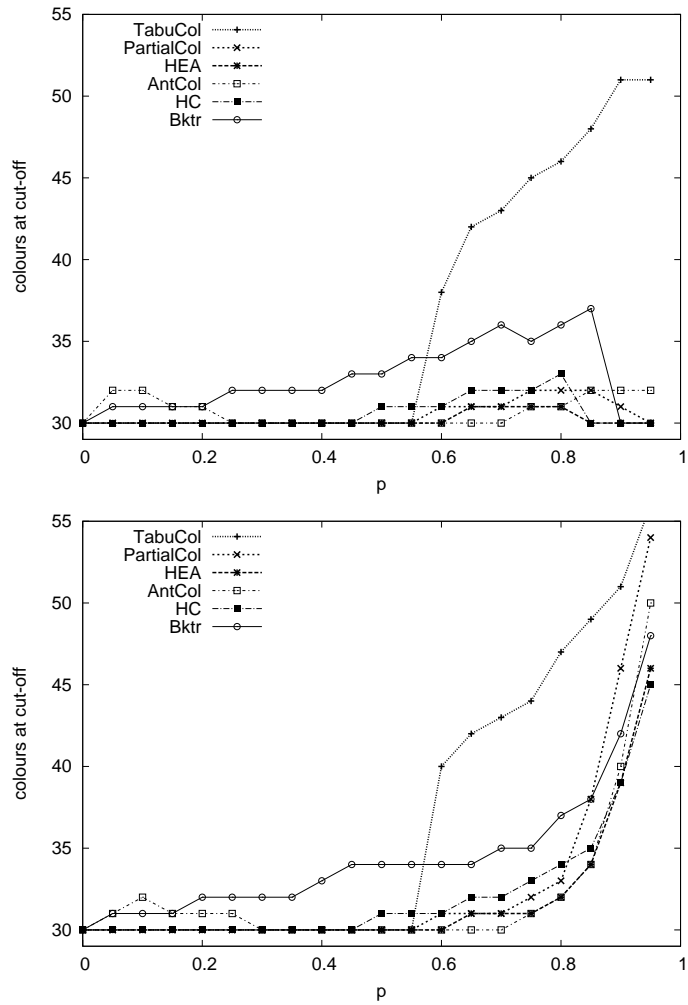


Figure 5.3: Summary of worst results from each algorithm with (top) “solvable” round-robin graphs of size  $|V| = 270$ , with  $\chi = 30$ ; and (bottom) round-robin graphs of size  $|V| = 270$ , with  $\chi \geq 30$ .



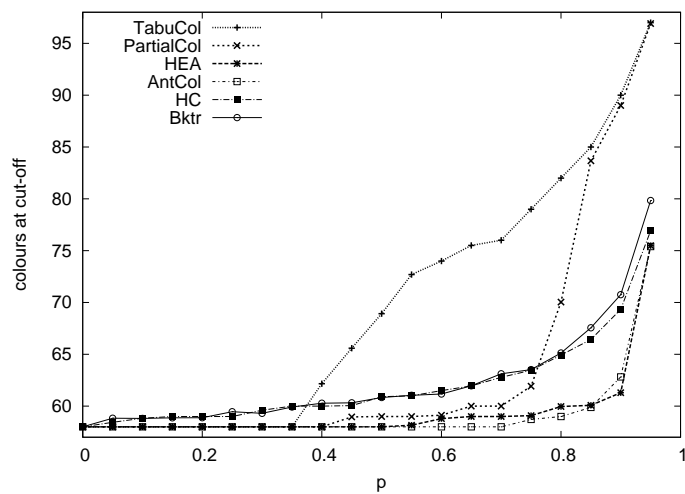
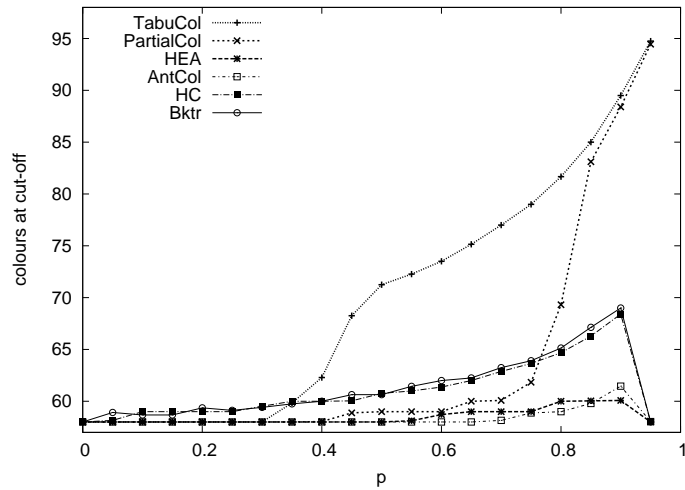


Figure 5.4: Summary of mean algorithm performance with (top) “solvable” round-robin graphs of size  $|V| = 928$ , with  $\chi = 58$ ; and (bottom) round-robin graphs of size  $|V| = 928$ , with  $\chi \geq 58$ .

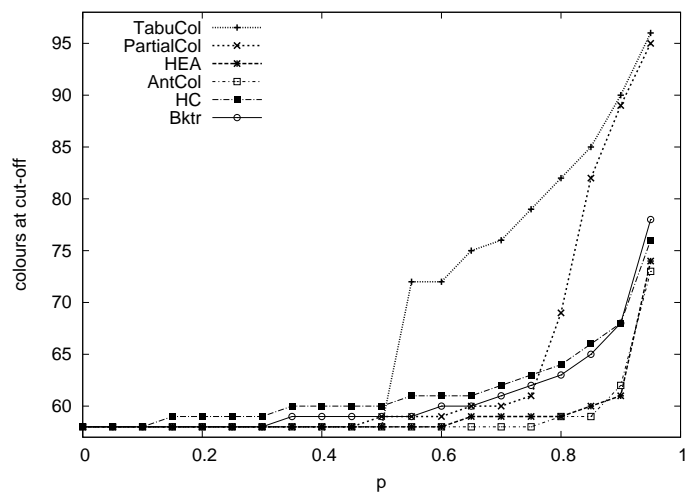
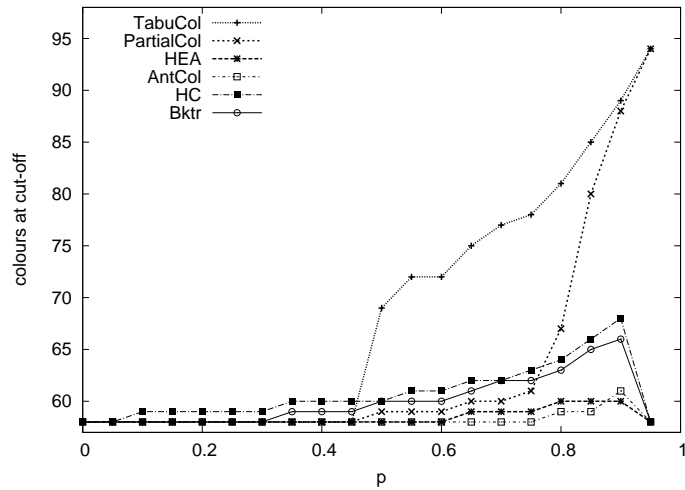


Figure 5.5: Summary of best results from each algorithm with (top) “solvable” round-robin graphs of size  $|V| = 928$ , with  $\chi = 58$ ; and (bottom) round-robin graphs of size  $|V| = 928$ , with  $\chi \geq 58$ .

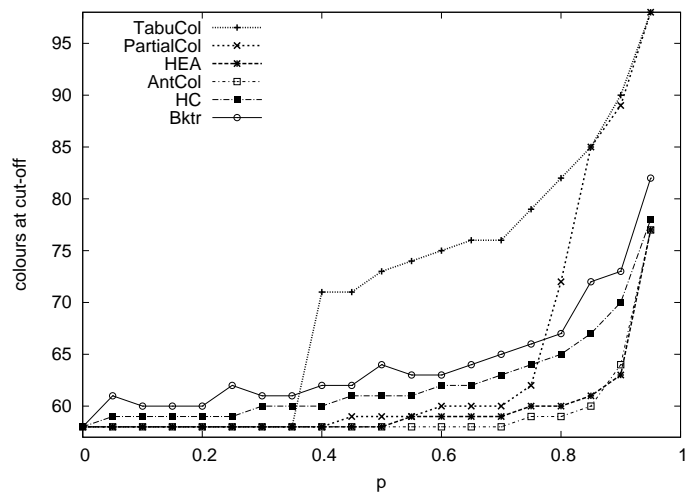
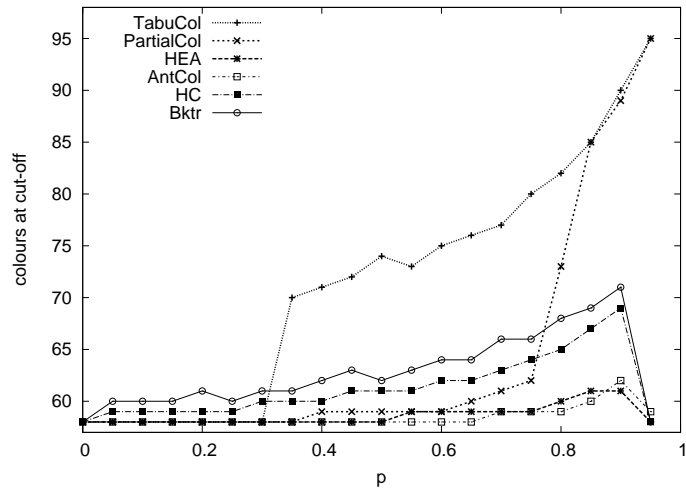


Figure 5.6: Summary of worst results from each algorithm with (top) “solvable” round-robin graphs of size  $|V| = 928$ , with  $\chi = 58$ ; and (bottom) round-robin graphs of size  $|V| = 928$ , with  $\chi \geq 58$ .

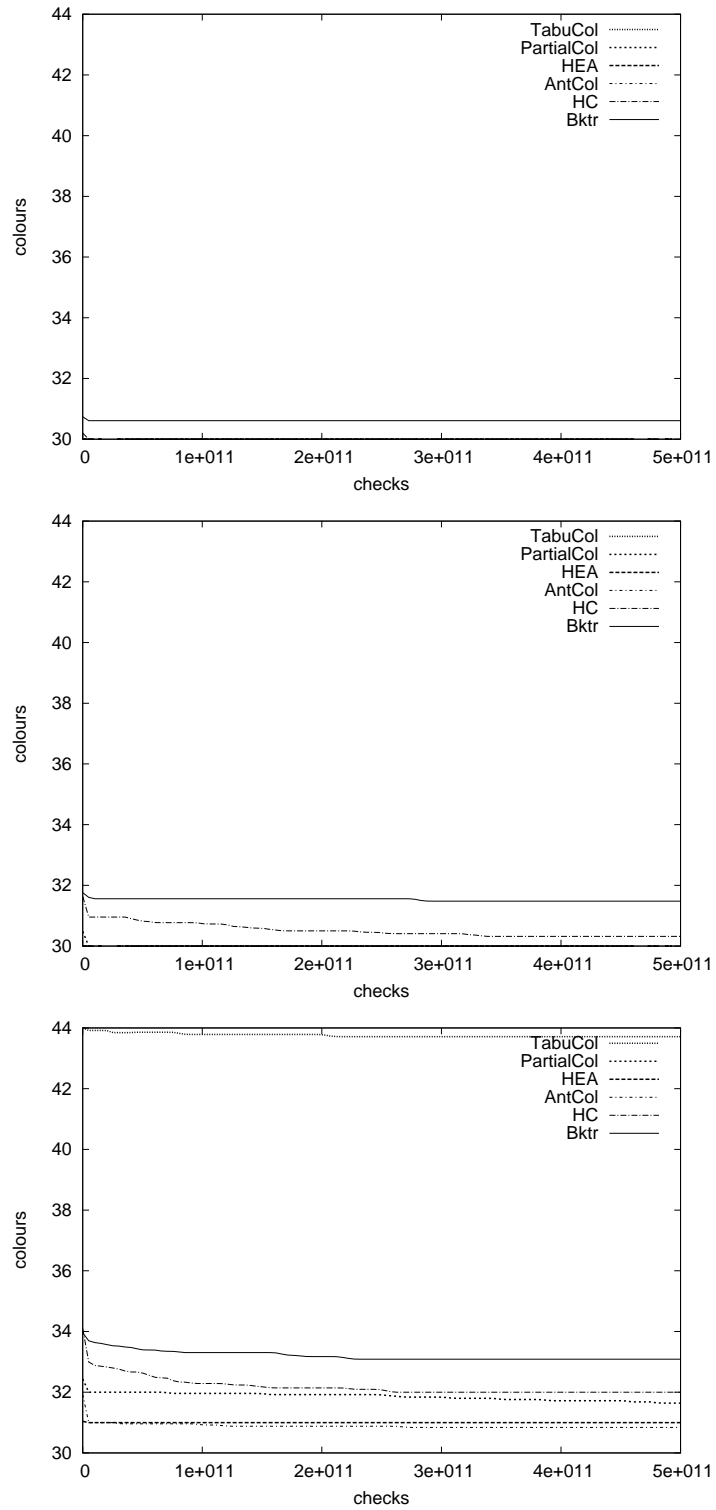


Figure 5.7: Run profiles for “solvable” round-robin graphs of size  $|V| = 270$ , with  $\chi = 30$ , for  $p = 0.25, 0.5$  and  $0.75$  respectively.

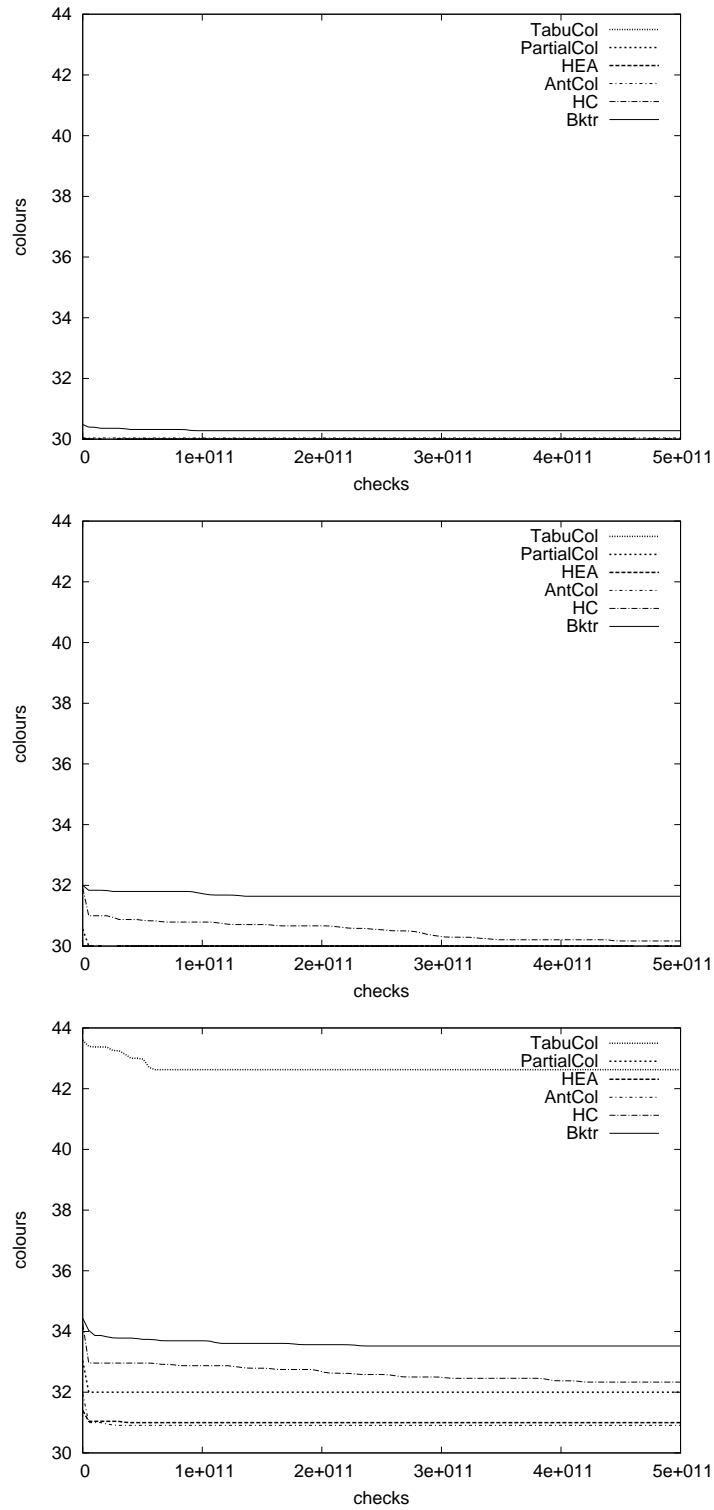


Figure 5.8: Run profiles for round-robin graphs of size  $|V| = 270$ , with  $\chi \geq 30$ , for  $p = 0.25$ , 0.5 and 0.75 respectively.

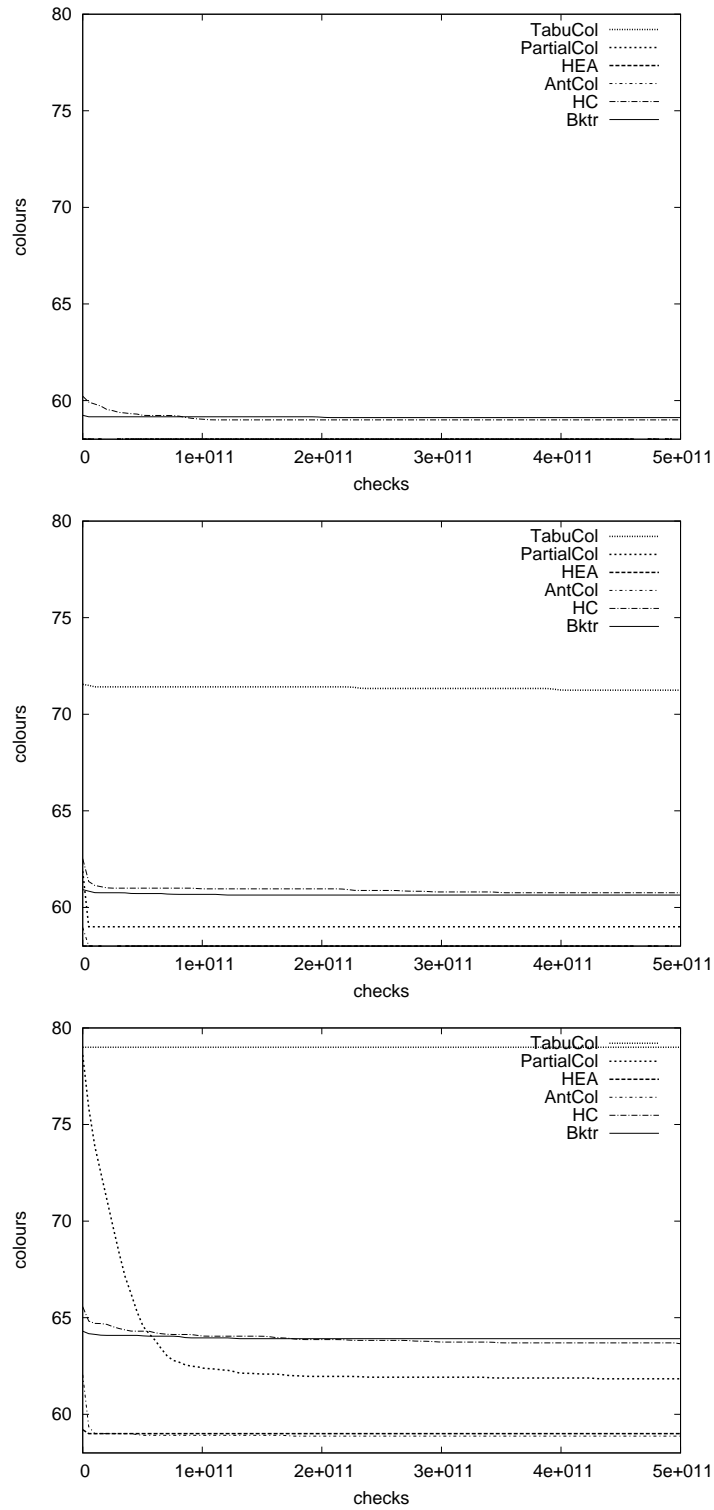


Figure 5.9: Run profiles for “solvable” round-robin graphs of size  $|V| = 928$ , with  $\chi = 58$ , for  $p = 0.25, 0.5$  and  $0.75$  respectively.

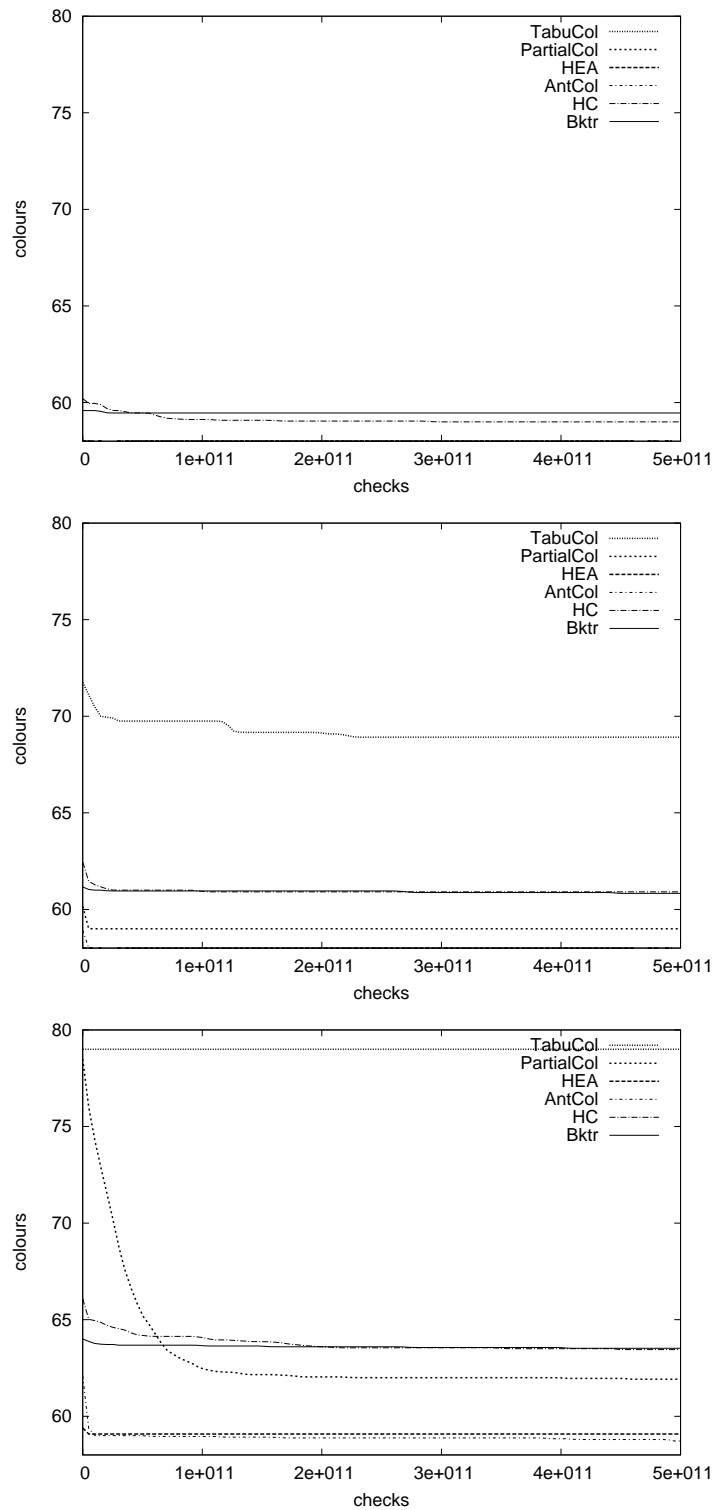


Figure 5.10: Run profiles for ‘round-robin graphs of size  $|V| = 928$ , with  $\chi \geq 58$ , for  $p = 0.25$ , 0.5 and 0.75 respectively.

# Chapter 6

## Graph Generation

Here we provide details on how to obtain/reproduce the various graph colouring problem instances used in our comparisons. Due to copyright issues we are unable to provide direct URLs to some of the sources, and in these cases we thus provide instructions for interested parties to follow. For space reasons we are also unable to provide exact copies of the artificially generated instances used (a zipped copy of all the  $|V| = 1000$  random instances, for example, occupies nearly 600MB). Thus, we provide details of the seeds and generator used instead.

### 6.1 Artificial Instances

The 1425 random graphs and 2060 flat graphs used in the study were constructed using the problem generator of Joe Culberson, download at:

<http://webdocs.cs.ualberta.ca/~joe/Coloring/Generators/generate.html>

The parameters and seeds for the random graphs were as follows (using  $|V| = 250, 500$  and  $1000$ ).

```
for (p=0.05; p<=0.95; p=p+0.05) do
  for (s=1; s<=25; s++) do
    Make random graph with  $|V|$ , seed  $s$ , and edge prob.  $p$ 
```

The parameters and seeds for the flat graphs were as follows (using  $|V| = 500$ ).

```
for (p=0.02; p<=0.22; p=p+0.005) do
  for (s=1; s<=20; s++) do
    Make flat graph with  $|V|$ , 10 partitions, seed  $s$ , edge prob.  $p$ , flatness 0
```

```
for (p=0.4; p<=0.8; p=p+0.02) do
  for (s=1; s<=20; s++) do
    Make flat graph with  $|V|$ , 50 partitions, seed  $s$ , edge prob.  $p$ , flatness 0
```

```
for (p=0.76; p<=0.96; p=p+0.005) do
  for (s=1; s<=20; s++) do
    Make flat graph with  $|V|$ , 100 partitions, seed  $s$ , edge prob.  $p$ , flatness 0
```



## 6.2 Real-World Instances

The thirteen timetabling problem instances used in the paper are due to the work of Mike Carter and can be downloaded at:

`ftp://ftp.mie.utoronto.ca/pub/carter/testprob/all_file.zip`

The round-robin graphs, meanwhile, were created using the instance generator of Rhyd Lewis, which can be downloaded at

`http://www.rhydLewis.eu/resources/RR-GColConverter.zip`

Finally, the social networking instances are due to the Add Health project:

`http://www.cpc.unc.edu/projects/addhealth`

Specifically, we used a random sample of twenty instances from the Add Health Network Structure files, whose original names are as follows (ordered in ascending size of  $|V|$ ):

#21, #38, #26, #29, #22, #30, #25, #32, #27, #33 for single-cluster instances;  
#4, #11, #7, #19, #14, #12, #13, #16, #15, and #17 for double-cluster instances.

These files can be obtained by contacting the Add Health project administrators.