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# A Sequence-based Selection Hyper-heuristic

## A Case Study in Nurse Rostering

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### 1 Introduction

Many effective search methodologies are tailored for a specific application. On the other hand, hyper-heuristics are a set of techniques that do not require any tailoring and operate on (and explore) the space of heuristics as opposed to directly searching the space of solutions [3]. The main goal is to raise the level of generality of search methods by offering a solver applicable to a wide range of domains rather than a single problem domain. Broadly, hyper-heuristics can be grouped into two classes: methodologies *selecting* from existing heuristics (e.g. mutation operations or hill climbers) or *generating* new heuristics to aid in solving hard computational problems. This work focuses on the selection type of hyper-heuristics. The reader is directed to [3] for a recent survey of hyper-heuristics.

Traditionally, a single point search selection hyper-heuristic framework employs two methods invoking them successively: a *selection method* to select a suitable low level heuristic from a suite of heuristics, applying that chosen heuristic to a candidate solution thereby generating a new one, and then a *move acceptance* method to decide whether to accept or reject the newly generated solution. In this study, the proposed method extends the former component of the traditional selection hyper-heuristic framework to choose a sequence of low level heuristics rather than a single low level heuristic at each decision point and then applies them consecutively to a candidate solution as if they were a single operator [5,6]. A challenge in the designing such a heuristic

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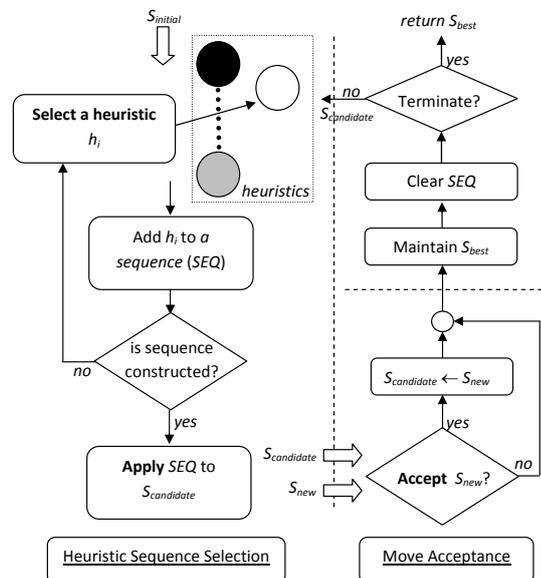


Fig. 1 A generic sequence-based selection hyper-heuristic framework

selection method is then to build an effective mechanism that can identify the best sequence of low level heuristics via learning during the search process.

The nurse rostering problem has been of interest to practitioners and researchers in the fields of operational research and artificial intelligence. This problem is known to be NP-hard [1]. We have joined the second international nurse rostering competition (INRC-II<sup>1</sup>) [4] to solve an extended version of the problem, referred to as the multi-stage nurse rostering problem, using a sequence-based selection hyper-heuristic method. The full description of the problem can be found at the competition website. We present our solution method in this study.

## 2 Sequence-based Selection Hyper-heuristic

A selection hyper-heuristic is a high level search methodology that operates on top of the traditional heuristics (or neighbourhood move operators). The combination of these simple operators to build more complex sequence of operators is the logic behind the development of a sequence-based hyper-heuristic framework. Figure 1 illustrates how a sequence-based selection hyper-heuristic framework operates.

The proposed solution method uses the generic sequence-based selection hyper-heuristic framework as a basis, aiming at analysing and producing sequences of heuristics during the optimisation process with a hidden Markov model (HMM) [2]. In HMM, hidden states are replaced with low level heuristics

<sup>1</sup> <http://mobiz.vives.be/inrc2/>

and observations are replaced with sequence-based acceptance strategies (AS). The method utilises two matrices: a transition score matrix ( $T_{Matrix}[\ ][\ ]$ ) to determine the movement between these states and an emission score matrix ( $AS_{Matrix}[\ ][\ ]$ ) to determine whether the selected sequence of heuristics will be applied to a current solution ( $AS = 1$ ) or will be coupled with another low level heuristic to form a sequence of heuristics ( $AS = 2$ ). Assuming the following set of low level heuristics  $H = \{h_1, h_2, \dots, h_n\}$ , all HMM matrices are initially assigned to 1. Hence, for any given heuristic ( $h_i$ ) and using a roulette wheel selection strategy, the probability to move to any other heuristic ( $h_j$ ) is  $1/n$ , initially. Moreover, the probability to select any of the sequence-based acceptance strategies is  $1/2$ , initially. Whenever a sequence of heuristics improves the quality of the best found solution, the relevant HMM scores get updated and increased by 1. Consequently, during the optimisation, the sequence-based hyper-heuristic adapts itself detecting a list of ‘promising’ sequences of heuristics that perform well. At any given step, the probability of moving from  $h_i$  to  $h_j$  is given by the following formula:

$$\frac{T_{matrix}[h_i][h_j]}{\sum_{\forall k}(T_{matrix}[h_i][h_k])} \quad (1)$$

The probability of selecting the acceptance strategy  $l$  of  $h_i$  is given by:

$$\frac{AS_{matrix}[h_i][l]}{\sum_{\forall k}(AS_{matrix}[h_i][k])} \quad (2)$$

The proposed sequence-based selection hyper-heuristic (SSHH) method mixes and controls a set of nine ‘simple’ low level heuristics to solve the instances of the second international nurse rostering competition. The results of the challenge reveal that the method is the most reliable one in terms of producing feasible solutions, and ranked third in overall competing against 15 other methods. The details of the methods and the results will be provided at the conference.

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